Paradoxes of special relativity & quantum mechanics

Today we are turning from metaphysics to physics. As we'll see, certain paradoxes about the nature of space and time result not from philosophical speculation, but from theories constructed in the physical sciences in response to experimental data.

We will be talking briefly about two of our most fundamental, and well-confirmed, theories of the physical world: the special theory of relativity and quantum mechanics.

GIven this topic, the presentation of the theories will be pretty superficial. The point will just be to present enough material for you to understand why the theories seem to lead to the paradoxes they do. The readings linked from the course web page go into more depth for those who would like to understand more of the science.

Let's begin with Einstein's theory of special relativity.

Let's begin with Einstein's theory of special relativity.

Einstein's theory is, for our purposes, an especially interesting one, because one can think of it as having its origins in a kind of paradox. (Einstein himself presents it that way in the reading on the web site.)

This paradoxes arises from the following three plausible, but jointly inconsistent, claims:

Galilean relativity: for any two objects moving at any speeds, their speeds relative to each other is the difference between their speeds if they're moving in the same direction, and the sum of their speeds if they are moving in the opposite direction.

The speed of light is a law of nature. (We'll follow convention by referring to this speed as "c".)

The principle of relativity: the laws of nature are the same in distinct frames of reference.

Each of these claims seems quite plausible on its own. But, as Einstein points out, they can't all be true.

If a ray of light be sent along the embankment, we see from the above that the tip of the ray will be transmitted with the velocity c relative to the embankment. Now let us suppose that our railway carriage is again travelling along the railway lines with the velocity v, and that its direction is the same as that of the ray of light, but its velocity of course much less. Let us inquire about the velocity of propagation of the ray of light relative to the carriage. It is obvious that we can here apply the consideration of the previous section, since the ray of light plays the part of the man walking along relatively to the carriage. The velocity W of the man relative to the embankment is here replaced by the velocity of light relative to the embankment. w is the required velocity of light with respect to the carriage, and we have

$$w = c - v.$$

The velocity of propagation of a ray of light relative to the carriage thus comes out smaller than c.

Galilean relativity: for any two objects moving at any speeds, their speeds relative to each other is the difference between their speeds if they're moving in the same direction, and the sum of their speeds if they are moving in the opposite direction.

The speed of light is a law of nature. (We'll follow convention by referring to this speed as "c".)

The principle of relativity: the laws of nature are the same in distinct frames of reference.

Each of these claims seems quite plausible on its own. But, as Einstein points out, they can't all be true.

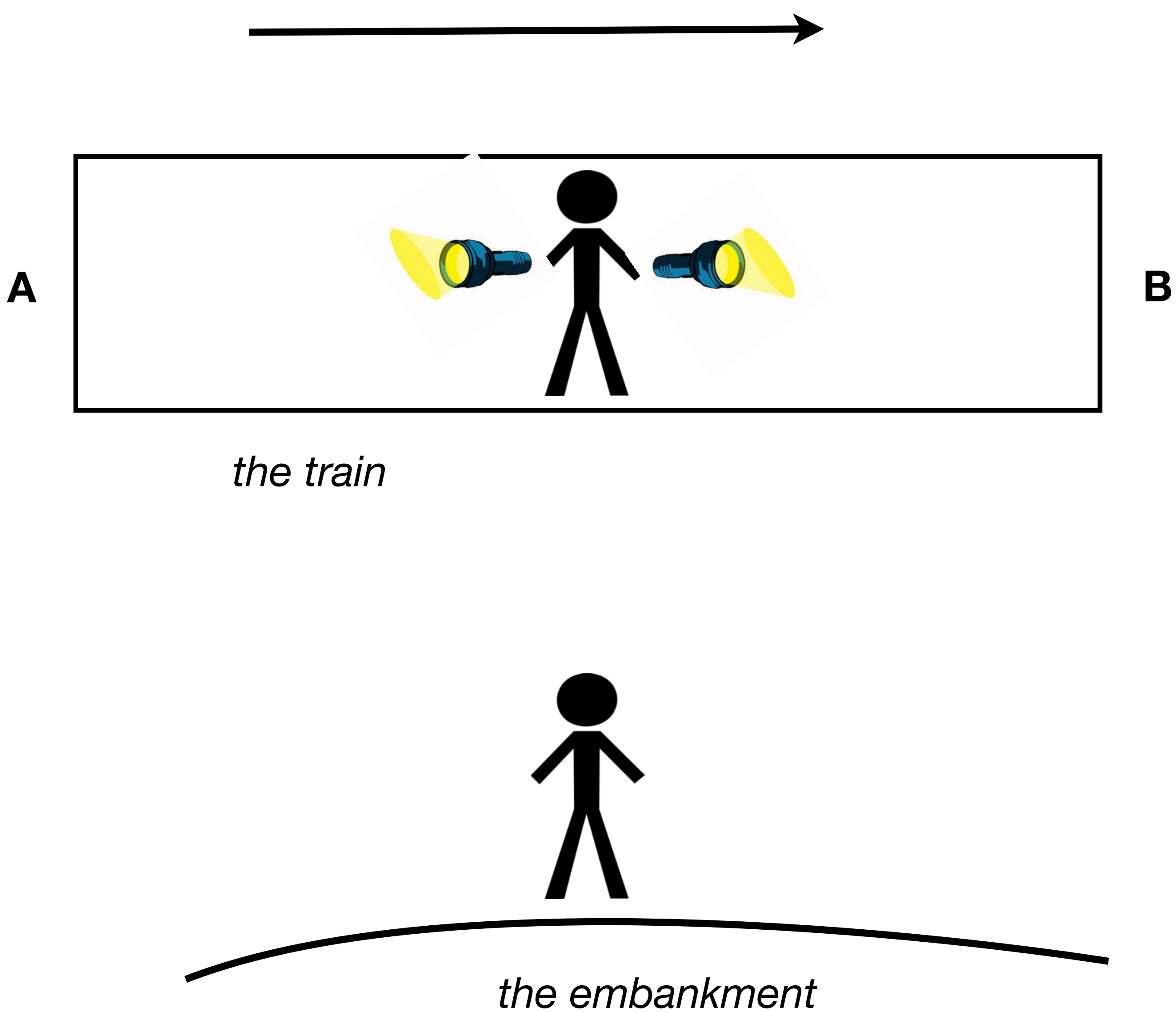
An initially plausible suggestion is that we should reject the claim that the speed of light is a law of nature, and say that the speed of light, like the speed of other things, can differ depending on one's speed relative to the light. But experiments designed to detect such differences in the speed of light failed to do so.

One of Einstein's innovations was to hold to the constancy of the speed of light while rejecting the principle of Galilean relativity.

However, this idea has some surprising consequences, which can be illustrated by example. (The example I use follows one Einstein also used in presenting his theory.)

The speed of light is a law of nature. (We'll follow convention by referring to this speed as "c".)

The principle of relativity: the laws of nature are the same in distinct frames of reference.



the train

the embankment

One of Einstein's innovations was to hold to the constancy of the speed of light while rejecting the principle of Galilean relativity.

However, this idea has some surprising consequences, which can be illustrated by example. (The example I use follows one Einstein also used in presenting his theory.)

Imagine two people, one in a train moving at a constant speed from left to right, and one on an embankment watching the train go by. We can imagine that the train is made of glass, so that the person on the embankment can see in.

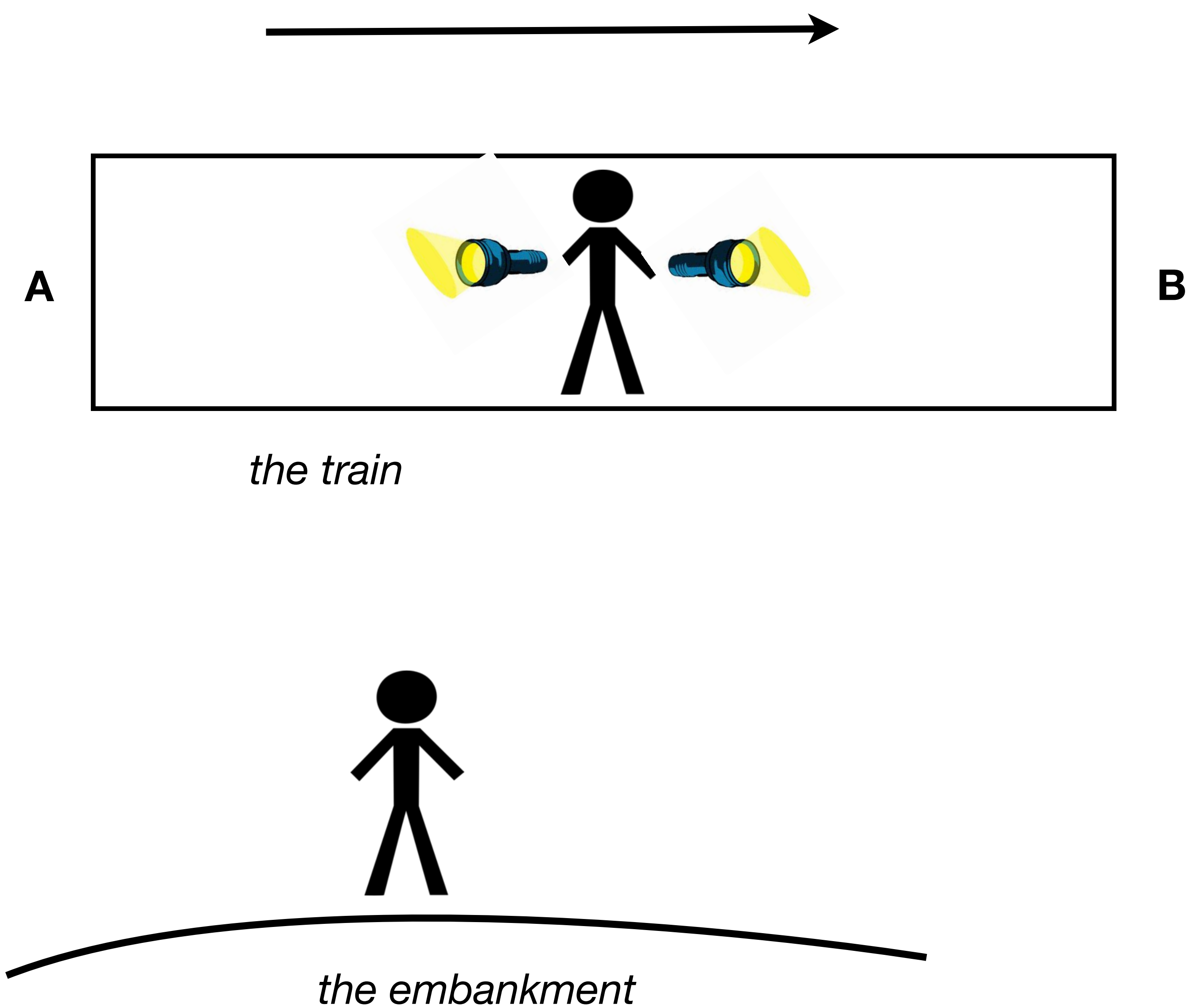
Now imagine that the person in the train car simultaneously turns on flashlights pointed at the two walls of the train car, A and B; and imagine further that he's at the exact midpoint of the train car.

Think about this situation first from the perspective of the person in the train car. Does the light reach A or B first?

But now think about this from the perspective of the person outside the train car. Do we get the same result?

The speed of light is a law of nature. (We'll follow convention by referring to this speed as "c".)

The principle of relativity: the laws of nature are the same in distinct frames of reference.



But now think about this from the perspective of the person outside the train car. Do we get the same result?

To see why not, it is important to note that the light takes some time to travel from the flashlights to the walls of the train, during which time the train travels some distance.

Hence it seems, looked at from the point of view of the person on the embankment, the location at which the left flashlight was turned on was closer to the location at which the light hits A than the location at which the right flashlight was turned on is to the location at which the light hits B.

But, given that the speed of both beams of light is the same from every frame of reference — including the person on the embankment — it follows that from his point of view the light hits A before it hits B. And this is not an illusion.

Hence, it seems, the light's hitting A is simultaneous with its hitting B relative to the frame of reference of the train, but not relative to the embankment.

The speed of light is a law of nature. (We'll follow convention by referring to this speed as "c".)

The principle of relativity: the laws of nature are the same in distinct frames of reference.

This can also be used to illustrate the phenomenon of **time dilation**. Intuitively, this is the phenomenon that if you are moving at a constant rate with respect to some frame of reference, time "speeds up" for you (slows down for them).

To see this, imagine the person on the train using a mirror opposite him in the car as a timekeeping device, which keeps time by the amount of time taken for light to reflect off of that mirror and back to him. Imagine again someone standing outside the train. Relative to someone outside the train, the light will be traveling further than for the person inside the train and hence (given the constancy of the speed of light) will take more time relative to the frame of reference outside the train.

The effect is that the "clock" constructed by the person inside the train will appear to be running slow. When their clock says that one second has passed, more than one second will have passed from the perspective of the frame of reference outside the train.

One of Einstein's innovations was to hold to the constancy of the speed of light while rejecting the principle of Galilean relativity.

Hence, it seems, the light's hitting A is simultaneous with its hitting B relative to the frame of reference of the train, but not relative to the embankment.

If simultaneity is relative to a frame of reference, so is **duration**. Consider the time between the flashlight being turned on and the beam of light hitting the back wall of the train car. This journey of the beam of light takes longer relative to the train car's frame of reference than relative to the frame of reference of the observer outside the train car.

The ordering of events can also change. Can you think of a variant of the above case in which one event happens before another from the perspective of the person on the train, but the ordering is reversed from the perspective of the frame of reference outside the train?

The speed of light is a law of nature. (We'll follow convention by referring to this speed as "c".)

The principle of relativity: the laws of nature are the same in distinct frames of reference.

This can also be used to illustrate the phenomenon of **time dilation**. Intuitively, this is the phenomenon that if you are moving at a constant rate with respect to some frame of reference, time "speeds up" for you (slows down for them).

To see this, imagine the person on the train using a mirror opposite him in the car as a timekeeping device, which keeps time by the amount of time taken for light to reflect off of that mirror and back to him. Imagine again someone standing outside the train. Relative to someone outside the train, the light will be traveling further than for the person inside the train and hence (given the constancy of the speed of light) will take more time relative to the frame of reference outside the train.

The effect is that the "clock" constructed by the person inside the train will appear to be running slow. When their clock says that one second has passed, more than one second will have passed from the perspective of the frame of reference outside the train.

But now suppose that the person outside the train has their own clock, of the same general sort. From the perspective of the person inside the train, will that clock be running slow, or fast?

This is a surprising result. One thinks that if A's clock is running fast relative to B's, then B's clock must be running slow relative to A's.

In fact, one might think that this is more than surprising; one might think that it is contradictory. After all, what would happen if A and B got together and compared watches? Surely each could not find that the other's watch was slow relative to their own.

This is a simple version of the **Twin Paradox**.

How this seeming paradox shows that the restriction to frames of reference in constant motion (neither accelerating or decelerating) is necessary.

The speed of light is a law of nature. (We'll follow convention by referring to this speed as "c".)

The principle of relativity: the laws of nature are the same in distinct frames of reference.

This can also be used to illustrate the phenomenon of **time dilation**. Intuitively, this is the phenomenon that if you are moving at a constant rate with respect to some frame of reference, time "speeds up" for you (slows down for them).

Time dilation, plus other aspects of the theories of special and general relativity which go well beyond anything I have presented, has convinced many that if these theories are true, then time travel should be possible.

As we will see, the possibility of time travel into the past gives rise to some puzzling questions. But you might have a question about this: if time travel really is possible, why have we not been visited by any time travelers?

One reason is that on some models of what time machines would be like, it is impossible to travel further back in time than the invention of such a machine, and such a machine has not yet been invented. Can you think of any other reasons?

The speed of light is a law of nature. (We'll follow convention by referring to this speed as "c".)

The principle of relativity: the laws of nature are the same in distinct frames of reference.

The real problems with time travel, though, may be brought out by the following sort of story:

> "You are very depressed. You are suicidally depressed. You have a gun. But you do not quite have the courage to point the gun at yourself and kill yourself in this way. If only someone else would kill you, that would be a good thing. But you can't really ask someone to kill you. That wouldn't be fair. You decide that if you remain this depressed and you find a time machine, you will travel back in time to just about now, and kill your earlier self. That would be good. In that way you even would get rid of the depressing time you will spend between now and when you would get into that time machine. You start to muse about the coherence of this idea, when something amazing happens. Out of nowhere you suddenly see someone coming towards you with a gun pointed at you. In fact he looks very much like you, except that he is bleeding badly from his left eye, and can barely stand up straight. You are at peace. You look straight at him, calmly. He shoots. You feel a searing pain in your left eye. Your mind is in chaos, you stagger around and accidentally enter a strange looking cubicle. You drift off into unconsciousness. After a while, you can not tell how long, you drift back into consciousness and stagger out of the cubicle. You see someone in the distance looking at you calmly and fixedly. You realize that it is your younger self. He looks straight at you. You are in terrible pain. You have to end this, you have to kill him, really kill him once and for all. You shoot him, but your eyesight is so bad that your aim is off. You do not kill him, you merely damage his left eye. He staggers off. You fall to the ground in agony, and decide to study the paradoxes of time travel more seriously."
> (Arntzenius & Maudlin, http://plato.stanford.edu/entries/time-travel-phys/)

The speed of light is a law of nature. (We'll follow convention by referring to this speed as "c".)

The principle of relativity: the laws of nature are the same in distinct frames of reference.

Could the time traveler in the story have succeeded in killing his former self? Why or why not?

Does it make sense to "go into the past and stop something from happening"?

What does all of this have to do with the TV show *Lost*?

More seriously, the residual puzzle here is a conflict between logical constraints on what time travelers could and could not do and our intuitive view of our own freedom of the will.

The tempting idea is that **if** we could go back in time, then surely we would then as now be free to do what we want; and surely this means that it is genuinely possible for us to do things we have the opportunity to do, such as killing our former selves. But this is **not** possible; hence either time travel must not be possible, or there would be some sort of odd asymmetry between our free will now and our free will post-time travel, or our views about the nature of our freedom of the will must be mistaken.

"You are very depressed. You are suicidally depressed. You have a gun. But you do not quite have the courage to point the gun at yourself and kill yourself in this way. If only someone else would kill you, that would be a good thing. But you can't really ask someone to kill you. That wouldn't be fair. You decide that if you remain this depressed and you find a time machine, you will travel back in time to just about now, and kill your earlier self. That would be good. In that way you even would get rid of the depressing time you will spend between now and when you would get into that time machine. You start to muse about the coherence of this idea, when something amazing happens. Out of nowhere you suddenly see someone coming towards you with a gun pointed at you. In fact he looks very much like you, except that he is bleeding badly from his left eye, and can barely stand up straight. You are at peace. You look straight at him, calmly. He shoots. You feel a searing pain in your left eye. Your mind is in chaos, you stagger around and accidentally enter a strange looking cubicle. You drift off into unconsciousness. After a while, you can not tell how long, you drift back into consciousness and stagger out of the cubicle. You see someone in the distance looking at you calmly and fixedly. You realize that it is your younger self. He looks straight at you. You are in terrible pain. You have to end this, you have to kill him, really kill him once and for all. You shoot him, but your eyesight is so bad that your aim is off. You do not kill him, you merely damage his left eye. He staggers off. You fall to the ground in agony, and decide to study the paradoxes of time travel more seriously." (Arntzenius & Maudlin, http://plato.stanford.edu/entries/time-travel-phys/)

Let's turn now to the second physical theory we'll be discussing today, quantum mechanics.

We'll be focusing on just one aspect of this theory, which is its use of the concept of **superposition.**

Albert (in the reading from *Quantum Mechanics and Experience* linked from the course web page) explains this concept using two sorts of properties of electrons, and I'll follow his development of the example.

He calls the two properties color and hardness, and these properties have three important characteristics:

1. They are **"on/off" properties** in the sense that there are exactly two hardnesses - hard and soft - and exactly two colors - black and white - and every electron has exactly one hardness and one color.

2. They are **independent**, in the sense that there is no correlation between the color of an electron and its hardness.

3. The properties are **measurable**, in the sense that we can test for (e.g.) the color of an electron and get the same result each time.

So far, so good. The oddities begin when we try to figure out **both** the color and hardness of an electron.

Suppose that we have a bunch of electrons, and measure the color of all of them. We then isolate the white ones. Now suppose we take this bunch of white electrons and measure their hardness. As expected, since color and hardness are independent in the above sense, we find that ½ of the white electrons are soft electrons and ½ are hard electrons. Now suppose we isolate the soft electrons from this bunch; it then seems that we will have a collection of electrons which all have color white and all have hardness of soft.

But we don't. If we re-measure the color of the electrons in the isolated bunch - all of which were previously measured to be white - we find that they are ½ white and ½ black.

He calls the two properties color and hardness, and these properties have three important characteristics:

1. They are **"on/off" properties** in the sense that there are exactly two hardnesses - hard and soft - and exactly two colors - black and white - and every electron has exactly one hardness and one color.

2. They are **independent**, in the sense that there is no correlation between the color of an electron and its hardness.

3. The properties are **measurable**, in the sense that we can test for (e.g.) the color of an electron and get the same result each time.

Suppose that we have a bunch of electrons, and measure the color of all of them. We then isolate the white ones. Now suppose we take this bunch of white electrons and measure their hardness. As expected, since color and hardness are independent in the above sense, we find that ½ of the white electrons are soft electrons and ½ are hard electrons. Now suppose we isolate the soft electrons from this bunch; it then seems that we will have a collection of electrons which all have color white and all have hardness of soft.

But we don't. If we re-measure the color of the electrons in the isolated bunch - all of which were previously measured to be white - we find that they are ½ white and ½ black.

It seems that something about measuring the hardness of the electrons changes their color. This is by itself not terribly surprising. The surprising thing is that the effect, and even the percentages, remain the same no matter how hardness is measured. (The exact opposite effects result from measuring color rather than hardness.)

The weirdness of this sort of effect is brought out nicely by the sort of experiment that Albert describes on pp. 8-11.

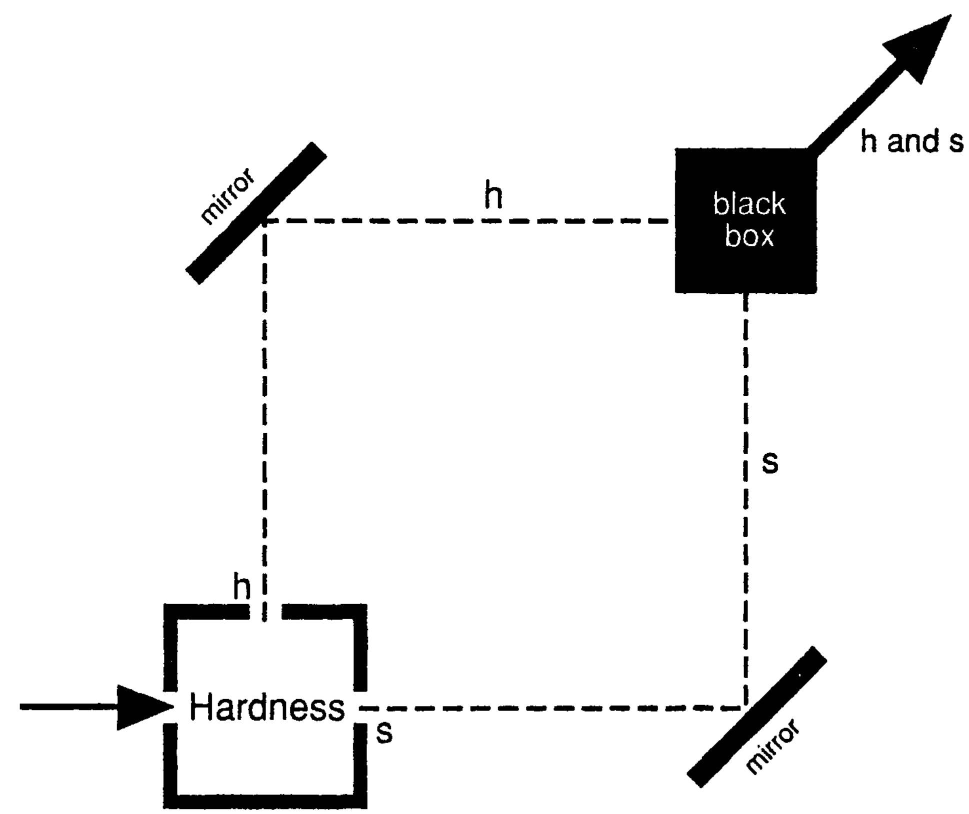
He calls the two properties color and hardness, and these properties have three important characteristics:

1. They are **"on/off" properties** in the sense that there are exactly two hardnesses - hard and soft - and exactly two colors - black and white - and every electron has exactly one hardness and one color.

2. They are **independent**, in the sense that there is no correlation between the color of an electron and its hardness.

3. The properties are **measurable**, in the sense that we can test for (e.g.) the color of an electron and get the same result each time.

This experiment is a sort of path through which electrons can be fed. They enter at lower left through a box which measures their hardness; if they are measured as soft, they are sent through the slit on the right side of the box, and if they are measured as hard, they are sent through the slit at the top side of the box. Both the "h" path and the "s" path terminate in the black box, through which all electrons exit on path "h and s."



Consider now what will emerge from the black box in the following cases:

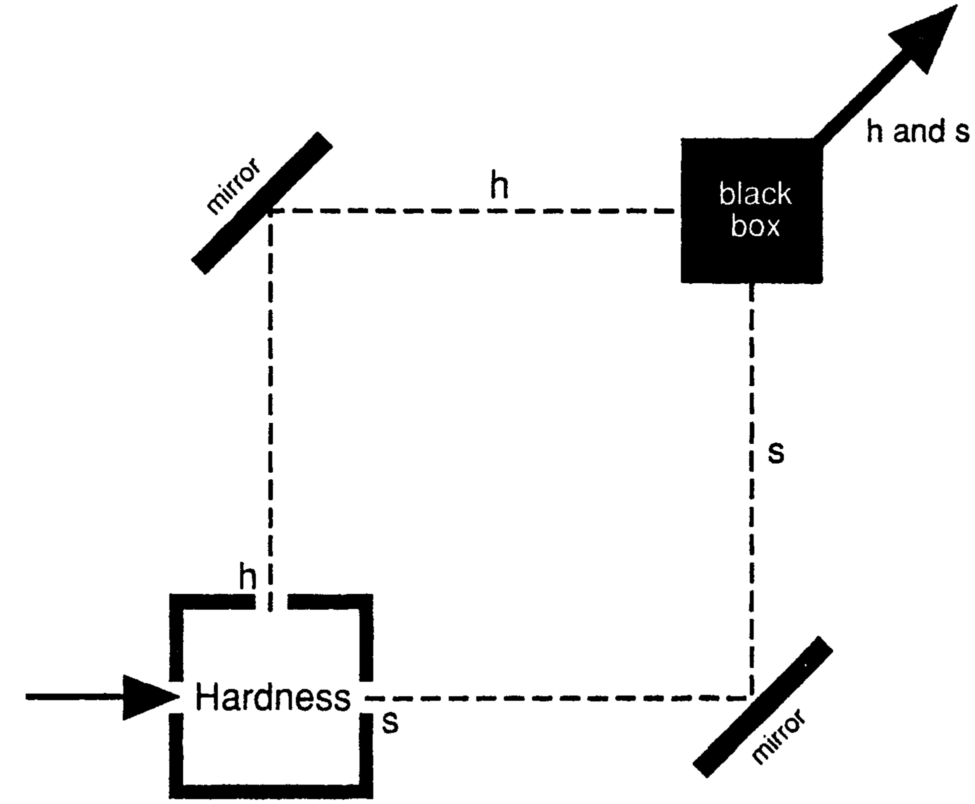
A stream of soft electrons are sent into the box.

A stream of hard electrons are sent into the box.

A stream of white electrons are sent into the box.

A stream of white electrons are sent into the box, with the "s" route blocked so that only the electrons which can take the "h" route exit from the box by route "h and s."

This experiment is a sort of path through which electrons can be fed. They enter at lower left through a box which measures their hardness; if they are measured as soft, they are sent through the slit on the right side of the box, and if they are measured as hard, they are sent through the slit at the top side of the box. Both the “h” path and the “s” path terminate in the black box, through which all electrons exit on path “h and s.”



Consider now what will emerge from the black box in the following cases:

A stream of soft electrons are sent into the box.

A stream of hard electrons are sent into the box.

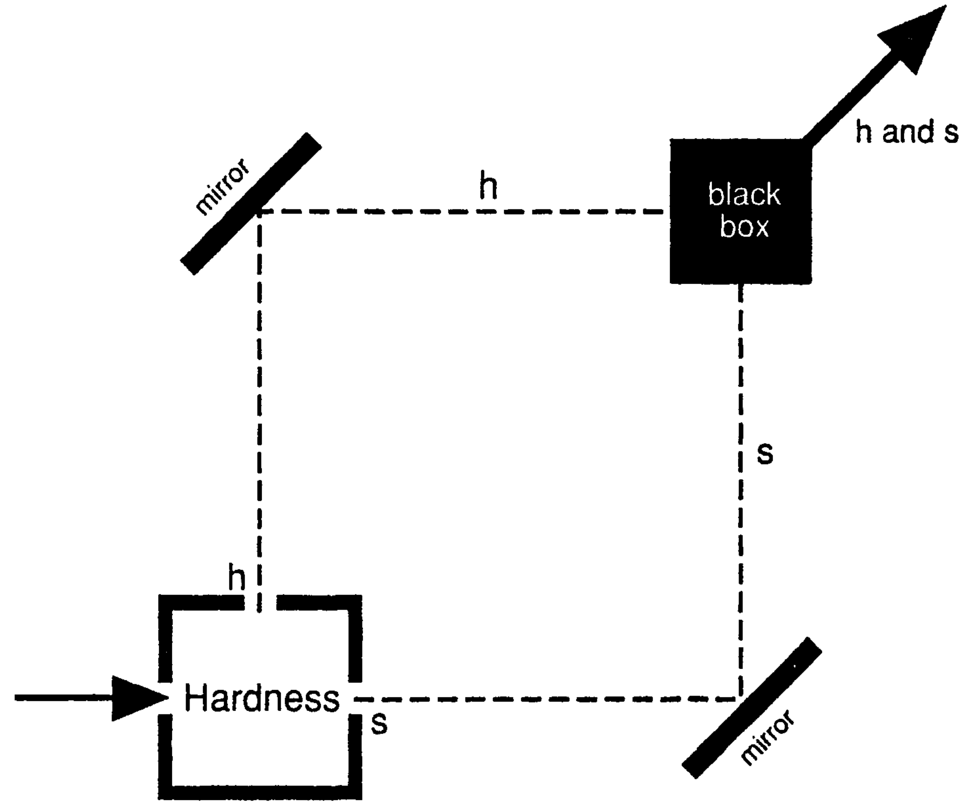
A stream of white electrons are sent into the box.

A stream of white electrons are sent into the box, with the “s” route blocked so that only the electrons which can take the “h” route exit from the box by route “h and s.”

Given the results in the fourth case, focus now on the third case, in which white electrons are fed into the box, and white electrons emerge along route “h and s.” Consider this question: how did those white electrons travel from the entrance to the exit?

A natural first thought is that since half of a collection of measured white electrons will be soft and half hard, ½ of the electrons traveled along route “h” and the other ½ traveled along route “s”. Why does this seem not to fit the fourth case described above?

A second idea is that each electron in some sense takes both routes; perhaps, for example, they split in half, with one half following the “h” route and one half following the “s” route until they rejoin at the black box. However, if we look at the paths to see what’s going on during the experiment, we never find divided electrons, or electrons somehow “spread out” between “h” and “s.” Every electron is always on one or the other path, but not both.



Given the results in the fourth case, focus now on the third case, in which white electrons are fed into the box, and white electrons emerge along route “h and s.” Consider this question: how did those white electrons travel from the entrance to the exit?

A natural first thought is that since half of a collection of measured white electrons will be soft and half hard, ½ of the electrons traveled along route “h” and the other ½ traveled along route “s”. Why does this seem not to fit the fourth case described above?

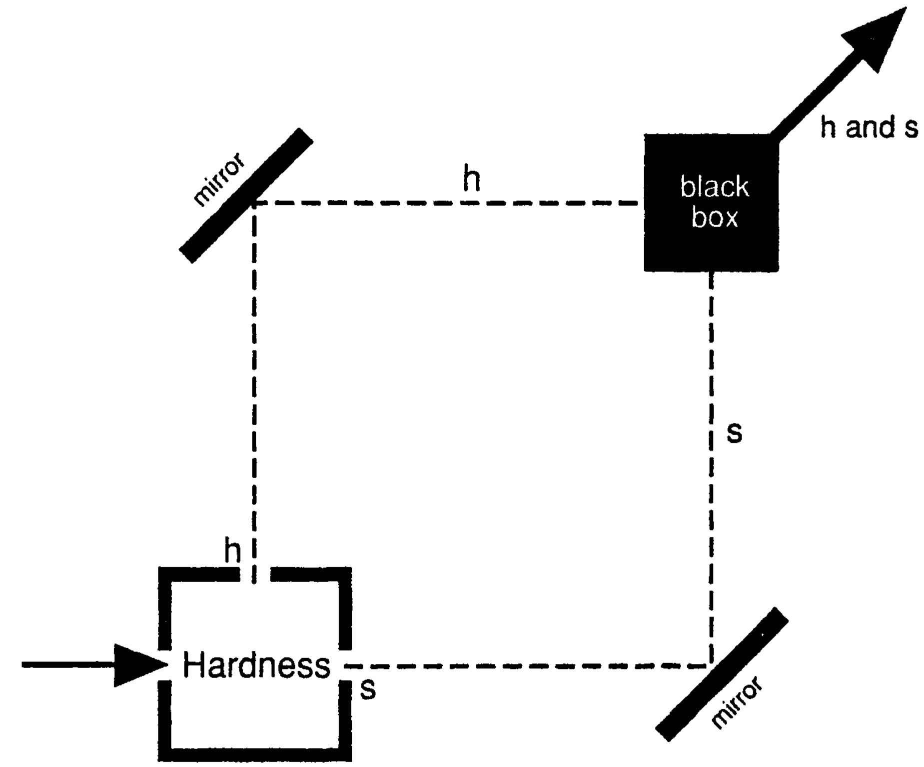
A second idea is that each electron in some sense takes both routes; perhaps, for example, they split in half, with one half following the “h” route and one half following the “s” route until they rejoin at the black box. However, if we look at the paths to see what’s going on during the experiment, we never find divided electrons, or electrons somehow “spread out” between “h” and “s.” Every electron is always on one or the other path, but not both.

Or maybe they took some third route. But what could this third route be? And why do we always find them on “h” or “s” when we check up on them?

This is thus some very puzzling experimental data. It can also be put in the form of an explicit paradox.

1. If a series of white electrons is sent through the box, all of them will still be white when they emerge along route ‘h and s’.
2. If a series of electrons moves from the entrance of the box to line ‘h and s’, one of the following must be true: (i) they all go along route ‘h’ or (ii) they all go along route ‘s’ or (iii) some go along route ‘h’ and the rest along route ‘s’ or (iv) some go by way of another route.
3. If we block both routes, no electrons arrive at the destination.
4. Option (iv) is false. (3)
5. If we block the ‘s’ route, the electrons which emerge are 1/2 white and 1/2 black.
6. If a series of electrons go through the box through the ‘h’ route, they will emerge 1/2 white and 1/2 black. (4,5)
7. Option (i) is false. (1,6)
8. If we block the ‘h’ route, the electrons which emerge are 1/2 white and 1/2 black.
9. If a series of electrons go through the box through the ‘s’ route, they will emerge 1/2 white and 1/2 black. (4,8)
10. Option (ii) is false. (1,9)
11. If a series of electrons are passed through the box, some of which go along the ‘s’ route and some of which go along the ‘h’ route, the electrons which emerge will be 1/2 white and 1/2 black. (6,9)
12. Option (iii) is false. (1,11)

C. No electrons move from the entrance of the box to line ‘h and s’. (2,4,7,10,12)



The conclusion of the argument is plainly false. Hence, if the argument is valid, it must have a false premise.

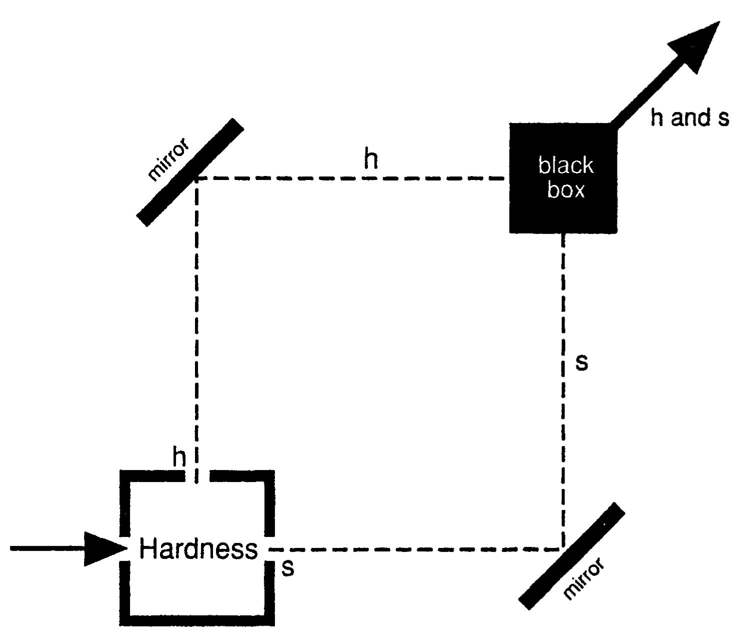
The only independent premises of the argument are 1, 2, 3, 5, and 8.

However, 1, 3, 5, and 8 are all experimentally verified, and hence are presumably not plausibly reject-able.

So if we grant that the argument is valid, the fault must lie with premise 2.

1. If a series of white electrons is sent through the box, all of them will still be white when they emerge along route 'h and s'.
2. If a series of electrons moves from the entrance of the box to line 'h and s', one of the following must be true: (i) they all go along route 'h' or (ii) they all go along route 's' or (iii) some go along route 'h' and the rest along route 's' or (iv) some go by way of another route.
3. If we block both routes, no electrons arrive at the destination.
4. Option (iv) is false. (3)
5. If we block the 's' route, the electrons which emerge are 1/2 white and 1/2 black.
6. If a series of electrons go through the box through the 'h' route, they will emerge 1/2 white and 1/2 black. (4,5)
7. Option (i) is false. (1,6)
8. If we block the 'h' route, the electrons which emerge are 1/2 white and 1/2 black.
9. If a series of electrons go through the box through the 's' route, they will emerge 1/2 white and 1/2 black. (4,8)
10. Option (ii) is false. (1,9)
11. If a series of electrons are passed through the box, some of which go along the 's' route and some of which go along the 'h' route, the electrons which emerge will be 1/2 white and 1/2 black. (6,9)
12. Option (iii) is false. (1,11)

C. No electrons move from the entrance of the box to line 'h and s'. (2,4,7,10,12)



So if we grant that the argument is valid, the fault must lie with premise 2.

If premise 2 is false, then the electrons get from the entrance to the exit without following "h", without following "s", without doing some of each, and without following some other path.

This is, admittedly, weird; but perhaps this is just the lesson to be absorbed from quantum mechanics. Perhaps there is some sense in which the electron is neither on one path or the other, and is not on both and is not on neither. This is part of what is meant by saying that the electron is in a state of **superposition** of being on route "h" and route "s."

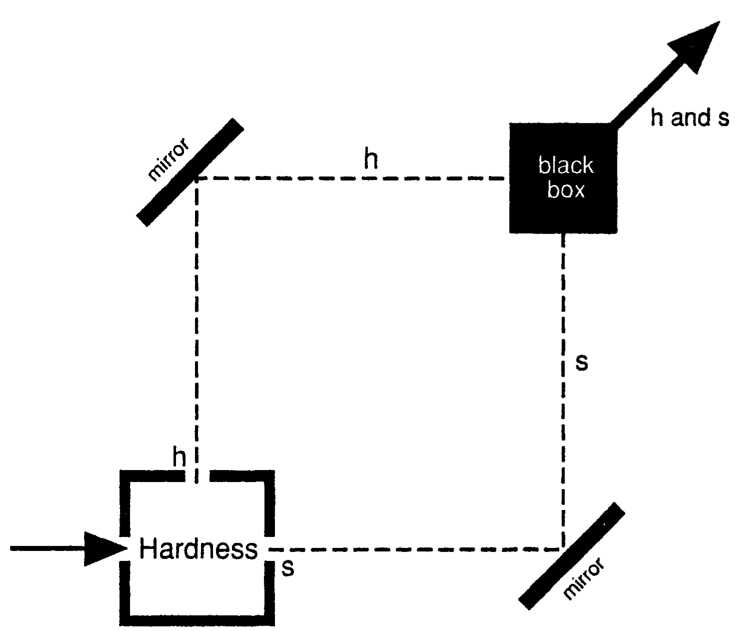
It is not easy to describe this state without contradicting itself; and even if one succeeds at avoiding contradiction, it is not obvious what sort of state one is describing.

But even if we grant that this makes sense, you might have the following puzzlement: how come we never observe the electron in such a state? How come when we check on the electron, is always doing something normal, like traveling along route "h"?

One answer to this question is that states of superposition **collapse** upon measurement. Perhaps the electron really was in this "neither here nor there" state, but this changes when we check up on it.

1. If a series of white electrons is sent through the box, all of them will still be white when they emerge along route 'h and s'.
2. If a series of electrons moves from the entrance of the box to line 'h and s', one of the following must be true: (i) they all go along route 'h' or (ii) they all go along route 's' or (iii) some go along route 'h' and the rest along route 's' or (iv) some go by way of another route.
3. If we block both routes, no electrons arrive at the destination.
4. Option (iv) is false. (3)
5. If we block the 's' route, the electrons which emerge are 1/2 white and 1/2 black.
6. If a series of electrons go through the box through the 'h' route, they will emerge 1/2 white and 1/2 black. (4,5)
7. Option (i) is false. (1,6)
8. If we block the 'h' route, the electrons which emerge are 1/2 white and 1/2 black.
9. If a series of electrons go through the box through the 's' route, they will emerge 1/2 white and 1/2 black. (4,8)
10. Option (ii) is false. (1,9)
11. If a series of electrons are passed through the box, some of which go along the 's' route and some of which go along the 'h' route, the electrons which emerge will be 1/2 white and 1/2 black. (6,9)
12. Option (iii) is false. (1,11)

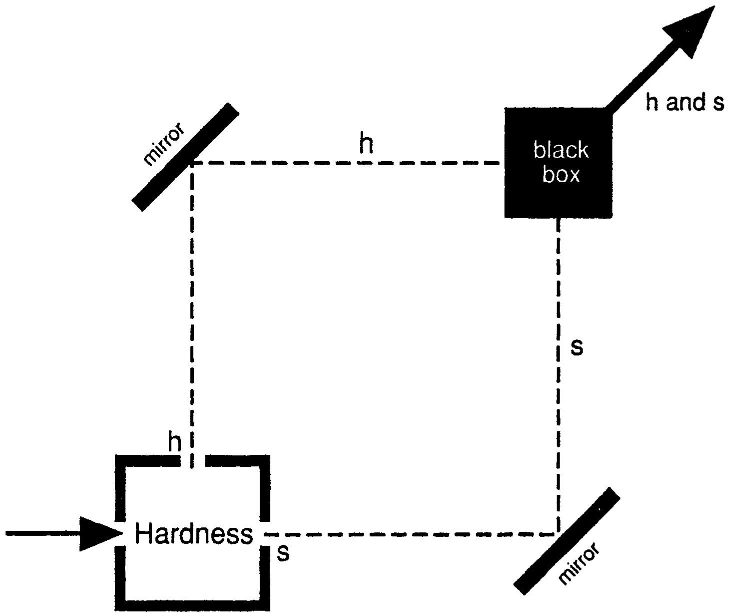
C. No electrons move from the entrance of the box to line 'h and s'. (2,4,7,10,12)

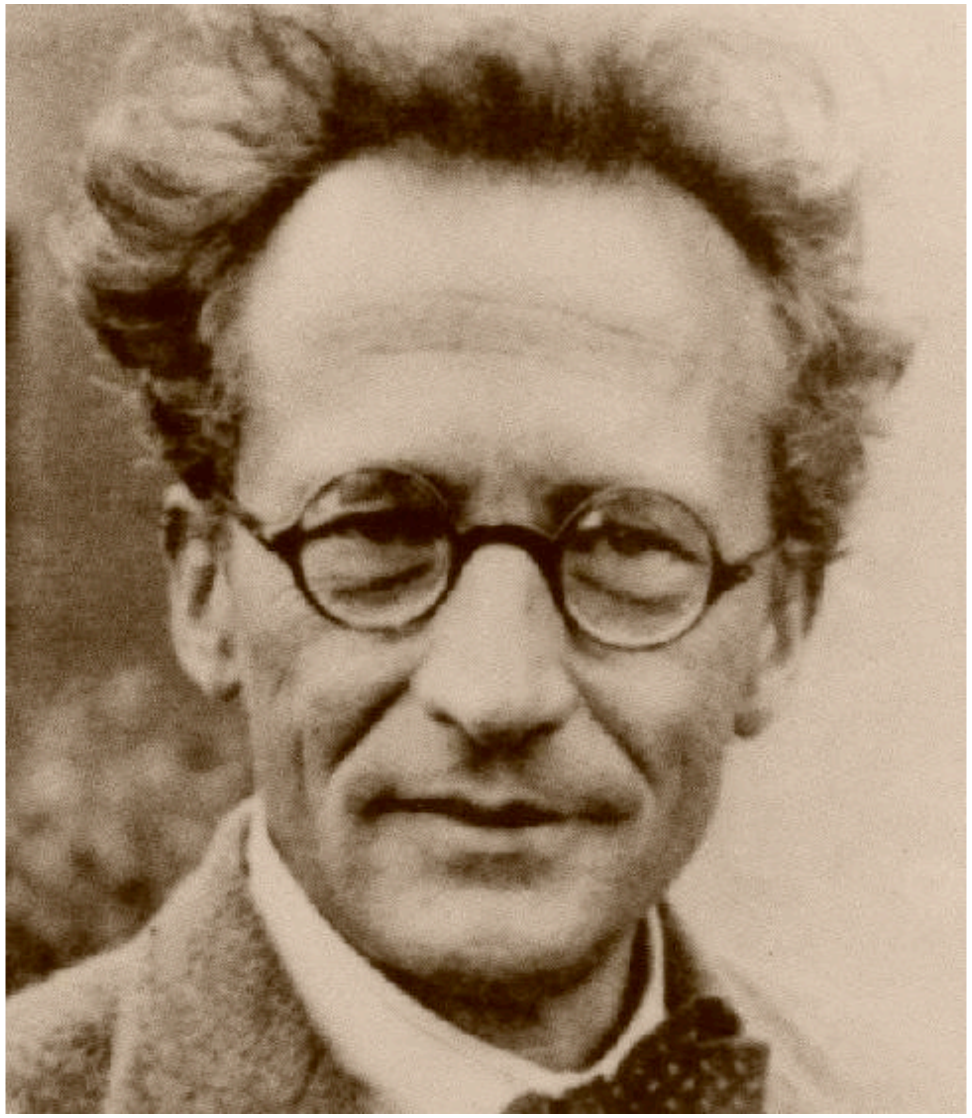


One answer to this question is that states of superposition **collapse** upon measurement. Perhaps the electron really was in this "neither here nor there" state, but this changes when we check up on it.

One might wonder: how does the electron know it is being watched? Many early proponents of quantum mechanics held the view that quantum collapse had something special to do with consciousness, though this is a minority position nowadays.

This idea - that electrons can be in states of superposition until being measured - perhaps seems acceptable so long as these states are confined to the world of the very small. But a famous puzzle due to the physicist Erwin Schodinger, one of the founders of quantum theory, casts doubt on whether this is really possible.





One answer to this question is that states of superposition **collapse** upon measurement. Perhaps the electron really was in this "neither here nor there" state, but this changes when we check up on it.

One might wonder: how does the electron know it is being watched? Why it is a mistake to think that the foregoing shows that collapse has anything special to do with consciousness.

This idea - that electrons can be in states of superposition until being measured - perhaps seems acceptable so long as these states are confined to the world of the very small. But a famous puzzle due to the physicist Erwin Schodinger, one of the founders of quantum theory, casts doubt on whether this is really possible.

"One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, *so* small, that *perhaps* in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives *if* meanwhile no atom has decayed. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts." (Schrödinger, 'The present situation in quantum mechanics', §5)

Could it make sense to say that the cat is in a state of superposition between being alive and dead?